MIND MAP : LEARNING MADE SIMPLE CHAPTER - 10

Straight Line

Parallel lines

1. When two lines are parallel their slopes are equal. Thus, any line parallel to $y = mx+c$ is of the type $y = mx +d$, where d is any parameter
2. Two lines $ax + by +c=0$ and $a'x + b'x + c' = 0$ are parallel if $\frac{a}{a'}=\frac{b}{b'}\neq\frac{c}{c'}$
3. The distance between two parallel lines with equations $ax +by +c_1 = 0$ and $ax +by +c_2 = 0$ is $\left|\frac{c_1-c_2}{\sqrt{a^2+b^2}}\right|$

Perpendicular lines

1. When two lines of the slope m_1 & m_2 are at right angles, the Product of their slope is -1, i.e. , $m_1m_2 = -1$. Thus, any line perpendicular to $y=mx + c$ is of the form , $y = \frac{-1}{m}x+d$ where d is any parameter.
2. Two lines $ax + by +c =0$ and $a'x + b'y + c'=0$ are perpendicular if $aa' + bb' =0$. Thus, any line perpendicular to $ax + by +c =0$ is of the form $bx - ay+ k =0$, where k is any parameter.

Angle between two straight lines in terms of their slopes

If m_1 and m_2 are the slopes of two intersecting lines ($m_1m_2\neq-1$) and θ be the acute angle between them then $\tan\theta=\left|\frac{m_1-m_2}{1+m_1m_2}\right|$

Slope Formula

- If θ is the angle at which a straight line is inclined to be +ve direction of x-axis and $0^\circ\leq\theta<180^\circ$, $\theta\neq 90^\circ$, then the slope of the line, denoted by m, is defined by $m = \tan\theta$. If θ is 90°, m doesn't exist, but the line is parallel to y-axis. If $\theta=0^\circ$, then $m=0$ and the line is parallel to x-axis.
- If $A(x_1, y_1)$ and $B(x_2, y_2)$, $x_1\neq x_2$ are points on straight line, then the slope m of the line is given by $m = (y_1 - y_2/ x_1 - x_2)$

Area of Triangle

Area of triangle whose vertices are (x_1, y_1), (x_2, y_2) and (x_3, y_3) is $\frac{1}{2}|x_1(y_1 - y_2) +x_2(y_3 - y_1) + x_3 (y_1 - y_2)|$

Distance Formula

The distance between the points $A(x_1, y_1)$ and $B(x_2, y_2)$ is $\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$

Section Formula

The P(x, y) divided the line joining $A(x_1, y_1)$ and $B(x_2, y_2)$ in the ratio m:n, then $x=\frac{mx_2+nx_1}{m+n}$; $y_1=\frac{my_2+ny_1}{m+n}$

Note:
- If m/n is +ve, the division is internal, but if m/n is –ve, the division is external.
- If m = n, then P is the mid-point of the line segment joining A & B.

Reflection & foot of perpendicular a point about a line

1. The image of a point(x_1, y_1) about a line $ax + by+ c =0$ is:

$$\frac{x-x_1}{a}=\frac{y-y_1}{b}=-\frac{ax_1+by_1+c}{\sqrt{a^2+b^2}}$$

2. Similarly, foot of perpendicular from a point on the line is:

$$\frac{x-x_1}{a}=\frac{y-y_1}{b}=\frac{-(ax_1+by_1+c)}{\sqrt{a^2+b^2}}$$

Length of the perpendicular from a point on a line

The length of the perpendicular from $P(x_1, y_1)$ on $ax + by +c =0$ is: $\left|\frac{ax_1+by_1+c}{\sqrt{a^2+b^2}}\right|$

Equation of Straight Line in various forms

1. **POINT-SLOPE FORM**: $y - y_1 = m(x - x_1)$ is the equation of a straight line whose slope is 'm' and passes through the point (x_1, y_1).
2. **SLOPE INTERCEPT FORM**: $y = mx +c$ is the equation of a straight line whose slope is 'm' and makes an intercept c on the y-axis.
3. **TWO POINT FORM**: $y - y_1 = \frac{y_2-y_1}{x_2-x_1}(x-x_1)$ is the equation of a straight line which passes through (x_1, y_1) & (x_2, y_2).
4. **INTERCEPT FORM**: $\frac{x}{a}+\frac{y}{b}=1$ is the equation of a straight line which makes intercepts a & b on x and y axis respectively.
5. **NORMAL / PERPENDICULAR FORM**: $x\cos\alpha + y\sin\alpha = p$ (where $p > 0, \leq 0\ \alpha < 2\pi$) is the equation of a straight line where the length of the perpendicular from origin O on the line is p and this perpendicular makes an angle α with +ve x-axis.
6. **GENERAL FORM**: $ax +by+ c =0$ is the equation of a straight line in general form. In this case, slope of line $=-\frac{a}{b}$